Semi Global Cotoal Domination upon Edge Addition Stable Graphs

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ABSTRACT

Let G be a simple, finite and connected graph. A subset D of vertices of a connected graph G is called a semi global cototal dominating set if D is a dominating set for G and G^{sc} and <V-D> has no isolated vertices in G, where G^{sc} is the semi complementary graph of G. The semiglobal cototal domination number is the minimum cardinality of a semi global cototal dominating set of G and is denoted by $\gamma_{sgcot}(G)$. A graph G is said to be semi global cototal domination edge addition stable, γ_{sgcot}^{+} −stable for short, if addition of any edge to G does not change the semi global cototal domination number. On the other hand, a graph G is said to be semi global cototal domination edge addition critical, if addition of any edge to G changes the semi global cototal domination number. In this paper, we study the concepts of semi global cototal domination upon edge addition stable property for cycle and path graphs.
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KEYWORDS: *Global cototal domination number, semi global cototal domination number, semi global cototal domination edge addition stable*

1. INTRODUCTION

For graph theoretical terms we may refer [2] and for terms related to domination we refer [3]. In our study, we consider only simple, finite and undirected graphs. A set of vertices D in a graph G is a dominating set, if each vertex of G is adjacent to some vertices of D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. A dominating set D of a graph G is a global dominating set if D is also a dominating set of $\overline{G}$. The global domination number $\gamma_g(G)$ is the minimum cardinality of a global dominating set of G[11]. A dominating set D of a graph G is a cototal dominating set if the induced sub graph, <V-D> has no isolated vertices. The cototal domination number $\gamma_{cot}(G)$ is the minimum cardinality of a cototal dominating set of G[6]. A dominating set D of a graph G is a global cototal dominating set if D is both a global dominating set and a cototal dominating set. The global cototal domination number $\gamma_{gcot}(G)$ is the minimum cardinality of a global cototal domination set of G [8] . We call $D \subseteq V(G)$ a semiglobal cototal dominating set, if D is a dominating set for G and G^{sc} and <V-D> has no isolated vertices in G, where G^{sc} is the semicomplementary graph of G. The semiglobal cototal domination number is the minimum cardinality of a semiglobal cototal dominating set of G and is denoted by $\gamma_{sgcot}(G)$[9].

A graph is said to be semi global cototal domination edge addition stable or γ_{sgcot}^{+} −stable for short, if addition of any edge to G does not change the semi global cototal domination number. On the other hand, a graph is said to be semi global cototal domination edge addition critical, if addition of any edge to G changes the semi global cototal domination number. In this paper, we study the concepts of semi global cototal domination upon edge addition stable property for cycle and path graphs.

We need the following results to prove further theorems.

1. For the cycle C_n, $n \geq 6$

$$\gamma_{sgcot}(C_n)= \begin{cases} \frac{n}{3}, & n \equiv 0 \ (mod\ 3); \\ \left\lceil\frac{n}{3}\right\rceil, & n \equiv 1 \ (mod\ 3); \\ \left\lceil\frac{n}{3}\right\rceil + 1, & n \equiv 2 \ (mod\ 3). \end{cases}$$

2. For a path P_n on n vertices,

$$\gamma_{sgcot}(P_n)= \begin{cases} \frac{n}{3} + 2\,, & n \equiv 0 \ (mod\ 3); \\ \frac{n+2}{3}, & n \equiv 1 \ (mod\ 3); \\ \left\lceil\frac{n}{3}\right\rceil + 1, & n \equiv 2 \ (mod\ 3). \end{cases}$$

2. MAIN RESULTS

In this section we investigate semi global cototal domination edge addition stable property for cycles and paths.

Definition 2.1[7] G be a connected graph, then the semi complementary graph of G, denoted by G^{sc}, has the same vertex set as that of G and has edge set {uv / u,v ∈ V(G), uv $\notin E(G)$ and there is w ∈ V(G) such that uw,wv ∈ E(G)}.

Definition 2.2[13] A dominating set D ⊆ V(G) is called a semiglobal dominating set of G if D is a dominating set for G and G^{sc}, where G^{sc} is the semicomplementary graph of G . The semiglobal domination number is the minimum cardinality of a semiglobal dominating set of G and is denoted by γ_{sg}(G).

Definition 2.3[9] We call D ⊆ V(G) a semiglobal cototal dominating set, if D is a dominating set for G and G^{sc} and <V-D> has no isolated vertices in G, where G^{sc} is the semicomplementary graph of G. The semiglobal cototal domination number is the minimum cardinality of a semiglobal cototal dominating set of G and is denoted by γ_{sgcot}(G).

Definition 2.4[4] Let P be a graph parameter. A graph G is said to be P-edge addition critical if P (G+e) ≠ P (G) for an edge e = uv between two non-adjacent vertices u and v in G. A graph G is said to be P-edge addition stable if P (G+e) = P (G).

Definition 2.5 A graph is said to be semi global cototal domination edge addition stable or γ^+_{sgcot} −stable in short, if addition of any edge to G does not change the semi global cototal domination number. In other words a graph G is γ^+_{sgcot} − stable if γ_{sgcot}(G+e) = γ_{sgcot}(G) for any edge e ∉ E (G).

Result 2.6 Cycle graph C_4 is not γ^+_{sgcot} − stable.

Proof: Let $V(C_4) = \{v_0, v_1, v_2, v_3\}$. Let $D_1 = \{v_0, v_1\}$ be the minimal semi global cototal dominating set of C_4 with $|D_1| = 2$. Now add an edge between two non adjacent vertices at a distance 2. Now $D_2 = \{v_0, v_1, v_2, v_3\}$ be the minimal semi global cototal dominating set of C_4 + e with $|D_2| = 4$. The semi global cototal dominating set of C_4 is increased by adding an edge. Hence C_4 is not γ^+_{sgcot} − stable. ∎

Result 2.7 Cycle graph C_5 is not γ^+_{sgcot} − stable.

Proof: Let $V(C_5) = \{v_0, v_1, v_2, v_3, v_4\}$. Let $D_1 = \{v_0, v_1, v_4\}$ be the minimal semi global cototal dominating set of C_5 with $|D_1| = 3$. Now add an edge between two non adjacent vertices at a distance 2. Now $D_2 = \{v_0, v_1\}$ be the minimal semi global cototal dominating set of C_5 + e with $|D_2| = 2$. The semi global cototal dominating set of C_5 is decreased by adding an edge. Hence C_5 is not γ^+_{sgcot} − stable. ∎

Theorem 2.8 For $n \equiv 0 \pmod 3$, $n \geq 6$, the cycle C_n is not $\gamma^{+}_{sgcot} -$ stable.

Proof: Let $V(C_n) = \{ v_0, v_1, v_2, v_3, \ldots, v_{n-1} \}$ and

$E(C_n) = \{ v_i v_{i+1} / i = 0, 1, 2, \ldots, n-1\}$, subscript modulo n.

If $G = C_n$, $(n \geq 6)$ then $C_n^{sc} = \begin{cases} C_{\frac{n}{2}} \cup C_{\frac{n}{2}} \; if\ n\ is\ even; \\ C_n \quad\quad if\ n\ is\ odd. \end{cases}$

Let D_1 be a minimal semi global cototal dominating set of C_n. Then $i+3 \leq n$ is the least positive integer such that v_i, $v_{i+3} \in D_1$. Add an edge $v_i v_j$ such that $d(v_i, v_j) = 2$. Let D_2 be the minimal semi global cototal dominating set of $C_n + e$. Choose either v_i or v_j as the first vertex of the γ_{sgcot}-set of

$C_n + e$. Both v_i and $v_j \notin D_2$, as it affects the cototal property. Choose the vertices of D_2 such that the induced subgraph $\langle V-D_2 \rangle$ has no isolated vertices and D_2 is a dominating set of both G and G^{sc}. Now we observe that semi global cototal domination number D_2 is exceeding D_1.

$\gamma_{sgcot}(C_n) \neq \gamma_{sgcot}(C_n + e)$. Hence the cycle C_n is not $\gamma^{+}_{sgcot} -$ stable for $n \equiv 0 \pmod 3$, $n \geq 6$.■

Corollary 2.9 For $n \equiv 1,2 \pmod 3$, $n \geq 7$, cycle graphs C_n are $\gamma^{+}_{sgcot} -$ stable.

Proof: By using the labeling of C_n as in the above theorem, the minimal semi global cototal dominating set of C_n can be obtained by taking $n \equiv 1 \pmod 3$ then D_1 contains v_{3i} where $i = 0, 1, \ldots, \frac{n-1}{3}$ and $n \equiv 2 \pmod 3$ then D_2 has v_{3i} where $i = 0, 1, \ldots, \frac{n-2}{3}$ and also v_{n-1}. On adding an edge in C_n at a distance 2, the semi global cototal domination number is not affected in all these cases. Hence C_n is $\gamma^{+}_{sgcot} -$ stable when $n \equiv 1,2 \pmod 3$, $n \geq 7$. ■

Result 2.10 The path P_3 is $\gamma^{+}_{sgcot} -$ stable. ■

Theorem 2.11 For $n \equiv 0,2 \pmod 3$, $n \geq 5$ the Path P_n is $\gamma^{+}_{sgcot} -$stable .

Proof: Let P_n be the path of order $n \equiv 0,2 \pmod 3$, $V(P_n) = \{ v_0, v_1, v_2, v_3, \ldots, v_{n-1}\}$.

If $G = P_n$ ($n \geq 3$) then $G^{sc} = P_{\frac{n}{2}} \cup P_{\frac{n}{2}}$ if n is even

$= P_{\frac{n+1}{2}} \cup P_{\frac{n-1}{2}}$ if n is odd

Since D_1 is a semiglobal cototal dominating set in G, $i+3 \leq n$ is the least positive integer such that v_i, $v_{i+3} \in D_1$. D_1 must contain v_0 and v_{n-1}, the end vertex of P_n.

If $n \equiv 0 \pmod 3$ then D_1 contains v_{3i} where $i = 0, 1, \ldots, \frac{n-3}{3}$ and the vertices v_{n-2} and v_{n-1}. If $n \equiv 2 \pmod 3$ then D_1 has v_{3i} where $i = 0, 1, \ldots, \frac{n-2}{3}$ and also v_{n-1}.

Now we add an edge $v_i v_j$ such that $d(v_i, v_j) = 2$. Particularly we choose v_i to be the pendant vertex v_0 and v_j to be the support vertex at a distance 2. Let D_2 be the semi global cototal domination number of P_n+e. By adding an edge between non adjacent vertices v_0 and v_2 in P_n, the semi global cototal domination number is not changed. That is $|D_1| = |D_2|$ in all cases. Hence P_n is γ^{+}_{sgcot} $-$stable when $n \equiv 0,2 \pmod 3$, $n \geq 5$. ■

Theorem 2.12 For n ≡ 1 (mod 3), n≥4 the Path P_n is not γ^{+}_{sgcot} −stable .

Proof: Let P_n be the path of order n ≡ 1 (mod 3), n≥4. $V(P_n) = \{ v_0, v_1, v_2, v_3, \ldots, v_{n-1}\}$. Let D_1 be the semi global cototal dominating set of P_n. If n ≡ 1 (mod 3) then $D_1 = \{v_{3i} / \ i = 0, 1, 2, \ldots, \frac{n-1}{3}\}$. Let D_2 be the semi global cototal domination number of P_n + e. By adding an edge between the pendant and support vertex v_0 and v_j at a distance 2, the semi global cototal domination number is increased. Hence P_n is not γ^{+}_{sgcot} − stable for n ≡ 1 (mod3), n≥4 . ■

3. CONCLUSION

In this paper we have studied the concept of semi global cototal domination upon edge addition stable property for cycle and path graphs. We shall explore various domination parameters upon edge addition stable property for cycle and path graphs as a part of our future work.

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